Message Encoding and Decoding Through Chaos Modulation in Chaotic Optical Communications

S. Tang, H. F. Chen, S. K. Hwang, and J. M. Liu

***Abstract*—Chaos modulation for message encoding and decoding is investigated using two chaotic optical communication systems. With a message injected into the chaotic dynamics of the transmitter in chaos modulation, the complexity of the chaos is found to increase. Meanwhile, the synchronization quality is not influenced by the message encoding. The effect of channel noise is investigated and the bit error rate is calculated as a function of the signal-to-noise ratio for both systems.**

***Index Terms*—Chaotic communication, chaos modulation, message encoding and decoding, semiconductor lasers.**

I. Introduction

WITH THE success in chaos synchronization [1], chaotic communication is of great current interest because of potential advantages in efficiently utilizing the broad bandwidth of a communication channel and potential applications in secure communications. Different methods of message encoding and decoding in chaotic communications have been proposed, which can be classified into three major categories: chaos masking [2], chaos shift keying [3], and chaos modulation [4]–[6]. Chaos modulation was first proposed in chaotic communication systems using Chua's circuits and Lorenz systems [4], [6]. Since optical communication systems are of great advantage in pushing the bit rate into the Gb/s region, chaos modulation has also been implemented in chaotic optical communication recently using optoelectronic feedback systems with chaotic wavelength fluctuation [7].

While chaos masking and chaos shift keying both have limitations in maintaining perfect synchronization during message transmission, chaos modulation has been shown to be able to provide perfect synchronization all the time in the presence of variations of the message [6], [8], [9]. This is because, in chaos modulation, a message is also injected into the dynamics of the transmitter when it is encoded onto the chaotic waveform of the transmitter and is then sent to the receiver. Both the transmitter and the receiver are driven by the same force which includes the message; thus, the identity between the transmitter and the receiver is maintained in the presence of message encoding.

Since the message participates in the evolution of the dynamics of the transmitter in chaos modulation systems, the output chaotic state of the transmitter can be very complicated due to the random nature of the message, which is a desired characteristic for secure communications. In fact, in the attempts to attack chaotic communication systems by eavesdroppers, it is found that the systems utilizing chaos modulation are relatively difficult to attack compared with those using chaos masking or chaos shift keying [10]–[13]. The reason is that the chaotic trajectory becomes time- and message-dependent, which makes it very difficult to unfold by using established reconstruction techniques [10], [11], thus increasing the practicality of this approach in secure communications. Therefore, how the chaotic state of a nonlinear dynamical system changes with message modulation is a very important issue that, to our knowledge, has never been studied.

In this paper, we numerically investigate the implementation of chaos modulation using two chaotic optical communication systems: an optical injection system, which takes a period-doubling route to chaos, and a delayed optoelectronic feedback system, which generates chaotic optical pulses through a route of quasi-periodicity. In both systems, numerical simulations are carried out to analyze the complexity of the chaotic state with the presence of message encoding. Synchronization quality is evaluated as the amplitude of the message is increased. System bit-error-rate (BER) performance through the chaos modulation scheme for both systems is also calculated. The influence of channel noise is investigated and the bit error is found to be mainly from the synchronization error induced by the channel noise. The objective of this paper is to study the basic characteristics of message encoding and decoding using the chaos modulation scheme in chaotic optical communication systems. Security of a communication system is a subject that deserves separate studies. It is not addressed in this paper except for a few brief comments.

II. General Theory

A. Correlation Dimension

To investigate the effect of message encoding on the chaotic dynamics, the correlation dimension ν is used to characterize the complexity of chaotic states. The correlation dimension ν has been found to be a good characterization of a chaotic state [14], [15]. When ν is high, the complexity of the chaotic state is high. To calculate ν, the correlation integral $C(N, r)$ of a dynamical state is defined as [14]

$$C(N, r) = \frac{1}{N^2} \sum_{i,j=1}^{N} \theta(r - \|X_i - X_j\|) \qquad (1)$$

where θ is the Heaviside step function, X_i and X_j are vectors constructed from the time series of the dynamical state,

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N is the number of points in the time series, and r is a prescribed small distance. The local slope of the correlation integral $\log C(N, r)$[15] is first calculated as

$$\nu(r_i) = \frac{\log C(N, r_{i-1}) - \log C(N, r_{i+1})}{\log r_{i-1} - \log r_{i+1}}. \quad (2)$$

The correlation dimension ν is then found at the position where $\nu(r_i)$ shows a plateau.

In chaos modulation, the random variation of a message is involved in the dynamics of the message-encoded system, making the dynamics different from that of the system without message encoding. Encoding messages in such a system is equivalent to increasing the number of degrees of freedom. Therefore, the message-encoded system has its own characteristics and dynamics which are not the same as those of the system without message encoding. The estimate of the correlation dimension, shown in the following discussions and figures when the system is message-encoded, represents an indicator of how the complexity of the dynamics varies with the message amplitude.

B. Correlation Coefficient and Synchronization Error

As was discussed in Section I, when message encoding through a chaos modulation scheme changes the chaotic dynamics, it is possible to maintain the synchronization quality of the system because both the transmitter and the receiver remain identical in the presence of a message. Synchronization quality is usually demonstrated by correlation plots with the output of the transmitter versus that of the receiver. To be quantitative, correlation coefficient and synchronization error can be calculated. The correlation coefficient ρ is defined as [16]

$$\rho = \frac{\langle [X(t) - \langle X(t)\rangle][Y(t) - \langle Y(t)\rangle]\rangle}{\sqrt{\langle |X(t) - \langle X(t)\rangle|^2\rangle}\sqrt{\langle |Y(t) - \langle Y(t)\rangle|^2\rangle}} \quad (3)$$

where $X(t)$ and $Y(t)$ are the outputs of the transmitter and the receiver, respectively, and $\langle . \rangle$ denotes the time average. The correlation coefficient is bounded as $-1 \le \rho \le 1$. A larger value of $|\rho|$ means better synchronization quality. Synchronization error is another criterion usually used to measure the synchronization quality. There has not been a standard definition of synchronization error, but a common one is defined as

$$\zeta = \frac{\langle |X(t) - Y(t)|\rangle}{\langle |X(t)|\rangle}. \quad (4)$$

When the synchronization error ζ is small, it means the synchronization quality is high. In this paper, both ρ and ζ are calculated to check the synchronization quality.

C. Channel Noise and BER

In a chaotic communication system, once chaotic synchronization is achieved between the transmitter and the receiver, message encoding and decoding can be implemented. Since the additive white Gaussian noise (AWGN) channel constitutes the most basic component of a communication link, the investigation of the system performance under AWGN is of great importance. Different from that of a conventional communication system, the performance of a chaotic communication system strongly depends on the quality of synchronization. Channel noise does not only contaminate the signal in the process of transmission but also seriously influences the synchronization quality of the system by generating synchronization error. The synchronization error is generated because the identity of the driving forces to both the transmitter and the receiver is corrupted by the presence of channel noise. Therefore, the system performance strongly depends on the robustness of synchronization under the influence of channel noise, which can be very different for different systems as we will show later in our demonstration of two chaotic optical systems.

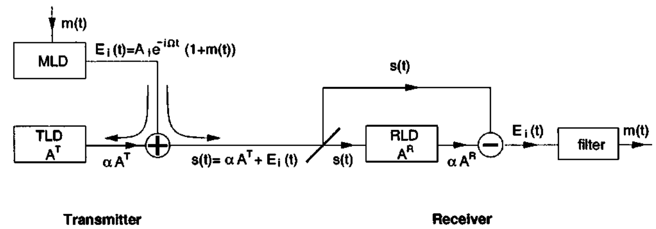

Fig. 1. Schematic setup of message encoding and decoding with additive chaos modulation in an optical injection system.

BER is a standard performance measurement of a communication system. It is often measured in a form as log BER versus signal-to-noise ratio (SNR). In a chaotic communication system, while additional energy has to be used to transmit the chaotic carrier waveform, the information is only included in the energy of the message. Therefore, it is common to treat the chaotic waveform as a carrier and include only the message energy in the calculation of the SNR. When the message is disturbed by noise, the signal levels designated to bits "1" and "0" spread out and overlap. Bit error is detected when a "1" bit is mistaken as a "0" bit and vice versa. As we have discussed above, the bit error caused by the channel noise comes from both the contamination of the message by the channel noise during transmission and the synchronization error generated by the injection of the channel noise into the receiver. As we will show later in Sections III and IV, the error from the second effect is very important for a chaotic communication system.

III. Optical Injection System

The configuration for message encoding and decoding through additive chaos modulation in an optical injection system is schematically shown in Fig. 1. In this system, an optical field $E_i(t) = A_i e^{-i\Omega t}$ from a master laser is injected into the transmitter laser to drive it into chaos through a period-doubling route [17], [18]. The parameter Ω denotes the frequency detuning between the transmitter laser and the master laser. A message $m(t)$ can be integrated onto the injection field $E_i(t)$ and then modulates the transmitter laser in several possible ways, including amplitude modulation, frequency modulation, and phase modulation. Amplitude modulation with $E_i(t) = A_i e^{-i\Omega t}(1 + m(t))$ is adopted in this paper.

According to the configuration in Fig. 1, the transmitter can be modeled by the following coupled equations in terms of the complex intracavity laser field amplitude A^T and the carrier density N^T[19], [20]

$$\frac{dA^T}{dt} = -\left(\frac{\gamma_c^T}{2} + \eta\alpha\right) A^T + i(\omega_0 - \omega_c)A^T + \frac{\Gamma}{2}\left(1 - ib^T\right) gA^T + F_{sp}^T + \eta s(t) \tag{5}$$

$$\frac{dN^T}{dt} = \frac{J}{ed} - \gamma_s N^T - \frac{2\epsilon_0 n^2}{\hbar\omega_0} g|A^T|^2 \tag{6}$$

where γ_c^T is the cavity decay rate, ω_0 is the free-oscillating frequency, ω_c is the longitudinal mode frequency of the cold laser cavity, Γ is the confinement factor, b^T is the linewidth enhancement factor, g is the optical gain coefficient including nonlinear effects, F_{sp}^T is the spontaneous emission noise source, η is the injection coupling rate, $s(t)$ is the transmitted signal, J is the bias current density, e is the electronic charge, d is the active layer thickness of the laser, γ_s is the spontaneous carrier decay rate, and n is the refractive index of the semiconductor medium [21].

The receiver, driven by the transmitted signal $s(t) = \alpha A^T(t) + E_i(t) = \alpha A^T(t) + A_i e^{-i\Omega t}(1 + m(t))$, with α being the coupling strength, can be described by

$$\frac{dA^R}{dt} = -\frac{\gamma_c^R}{2} A^R + i(\omega_0 - \omega_c)A^R + \frac{\Gamma}{2}\left(1 - ib^R\right) gA^R + F_{sp}^R + \eta s(t) \tag{7}$$

$$\frac{dN^R}{dt} = \frac{J}{ed} - \gamma_s N^R - \frac{2\epsilon_0 n^2}{\hbar\omega_0} g|A^R|^2. \tag{8}$$

Comparing (5)–(8), the transmitter and the receiver are identical and are driven by a common force $s(t)$. When the receiver laser is stably synchronized to the transmitter laser, $A^R(t) = A^T(t)$, the message can be recovered at the receiver end by recovering the message-carrying field as the following:

$$\begin{aligned} E_i^R(t) &= s(t) - \alpha A^R(t) \\ &= \alpha A^T(t) + E_i(t) - \alpha A^R(t) = E_i(t) \\ &= A_i e^{-i\Omega t}(1 + m(t)) \end{aligned} \tag{9}$$

and the message $m(t)$ is finally retrieved by passing $E_i(t)$ through a filter. Details on the laser modeling and the discussion on the synchronization condition can be found in [20].

When the encoding message $m(t)$ is injected into the laser dynamics of the transmitter as in (5), the chaotic state of the transmitter is modulated by the message. To investigate how message modulation influences the dynamics of the transmitter and further influences the system performance, numerical simulations are carried out based on (5)–(9). Typical intrinsic laser parameters as given in [20] are used in the simulations. The encoding message $m(t)$ used in the simulation is nonreturn-to-zero (NRZ) random digital bits "1" and "0", with the amplitude of "1" indicated by $m(t) = \varepsilon$ and that of "0" as $m(t) = 0$. The strength of the message can be varied by adjusting the parameter ε. The bit rate of $m(t)$ can be as high as several Gb/s as the chaotic output of the optical injected laser is very fast. In this paper, the bit rate is set at 2.5 Gb/s as a typical value.

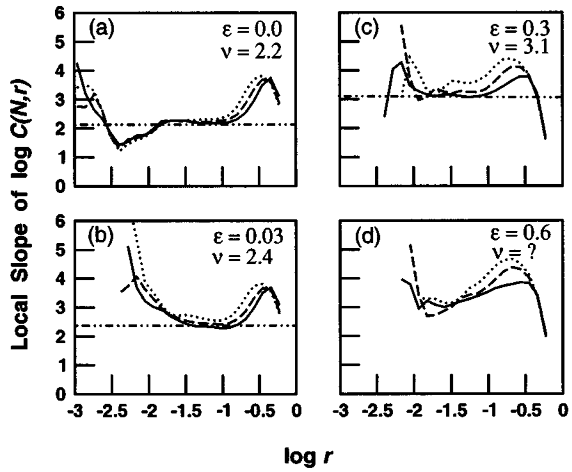

Fig. 2. Calculated correlation dimension plots with different message amplitudes in the optical injection system.

The influence of message encoding on the chaotic laser dynamics is first investigated. For the optical injection system, the chaotic output of the transmitter $A^T(t)$ is recorded as the time series and is used to analyze the correlation dimension ν. Fig. 2 shows the plots of the local slope of $\log C(N, r)$ versu$\log r$ for different message amplitudes. The three coincident curves in each plot correspond to the same local slope of $\log C(N, r)$ but are calculated with different embedding dimensions and delay times when the vector space is reconstructed as in (1). The plateau where the value of ν is retrieved is indicated by a horizontal line and the value is shown in each plot. When there is no message encoded as $\varepsilon = 0.0$ in Fig. 2(a), the correlation dimension is found to be $\nu = 2.2$, which is the characteristic of the original chaotic state of the optical injection system in the absence of chaos modulation. In the presence of an encoded message of amplitude $\varepsilon = 0.03$, the correlation dimension is increased to $\nu = 2.4$, as is shown in Fig. 2(b). In Fig. 2(c), when the message amplitude is increased to $\varepsilon = 0.3$, the correlation dimension is found to be larger than 3 with $\nu = 3.1$. Originally, the degrees of freedom of an optical injection system are 3 and the correlation dimension is capped to be below 3. Now, as we can see, due to the effect of message encoding, the dimension of the system is increased to be higher than 3. Finally in Fig. 2(d) when $\varepsilon = 0.6$, the correlation plot shows a characteristic of noise-like behavior where the local slope of the correlation integral has no flat part at all. It indicates that the chaotic state is now very complicated and the dimension of the system can be very high. This analysis indicates that the injection of a message into the dynamics of the transmitter increases the complexity of the chaotic state dramatically under certain situations. This makes it very difficult for an attacker to decode the message without knowing exactly all the parameters of the transmitter. In the analysis above, the intrinsic spontaneous emission noise of the laser is not included in order to clearly show the effect of message encoding on the chaotic dynamics. With the intrinsic noise of the laser, the chaotic state can become even more complicated [22].

Though message modulation can increase the complexity of the chaotic state, it does not break the identity between the transmitter and the receiver, as is shown in (5)–(8). When there is

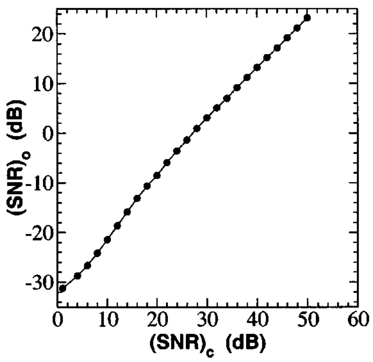

Fig. 3. Synchronization performance of the chaotic optical injection system indicated by $(SNR)_c$ versus $(SNR)_o$.

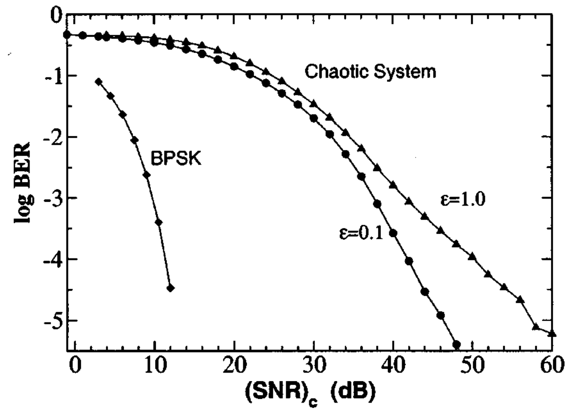

Fig. 4. BER versus $(SNR)_c$ of the chaotic optical injection system. The BER of a BPSK system is also shown for comparison.

no message encoded, the correlation coefficient between the chaotic outputs of the transmitter and the receiver is calculated to be $\rho = 0.998$ and the synchronization error is $\zeta = 0.04$. The imperfect synchronization is caused by the intrinsic spontaneous emission noise of the lasers. As the message amplitude increases from $\varepsilon = 0$ to 1.0, both the correlation coefficient and the synchronization error show no significant deterioration. This property holds even when some parameter mismatch exists between the transmitter and the receiver. Nevertheless, the amplitude of $m(t)$ cannot exceed the extent where the driving effect of $m(t)$ causes the output of the transmitter to lose the chaotic characteristics required by chaotic communication. Therefore, the amplitude of $m(t)$ is capped by the requirement for keeping the dynamics in the chaotic region.

Channel noise is an important effect in determining the system BER performance. In chaotic communications, the influence of channel noise is further amplified through the process of generating synchronization error. When there is no synchronization error, the message is just contaminated by the channel noise as $E_i(t) + n(t)$. When there is synchronization error, the recovered message is $E_i(t) + n(t) + \alpha A^T(t) - \alpha A^R(t)$. Therefore, the system performance directly depends on the characteristics of the synchronization error. The synchronization error is generated when channel noise is injected into the receiver dynamics and breaks the identity between the transmitter and the receiver. As the generation of synchronization error involves the nonlinear dynamics of a system, the synchronization error caused by channel noise varies significantly from one system to another.

In the optical injection system, when channel noise is injected into the receiver system through $A^{R(t)}$, enormous synchronization error or even desynchronization is observed. To show the significance of how channel noise generates synchronization error, the effect of channel noise is studied in the comparison of the channel SNR, denoted as $(SNR)_c$, to the receiver output SNR, denoted as $(SNR)_o$. In the definitions, $(SNR)_c = 10\log(P_m/{\sigma_n}^2)$ and $(SNR)_o = 10\log(P_m/{\sigma_o}^2)$, where P_m is the power of the transmitted message, ${\sigma_n}^2 = N_0/2T_b$ is the variance of the channel noise with $N_0/2$ being the power spectral density of the channel noise and T_b being the bit duration, and ${\sigma_o}^2$ is the variance of the total output noise which is defined as $n(t) + \alpha A^T(t) - \alpha A^R(t)$ with the synchronization error included. Fig. 3 shows the result of this comparison when the message amplitude is set at $\varepsilon = 0.1$. It is shown that $(SNR)_o$ is about 27 dB worse than $(SNR)_c$ because of the synchronization error. The dependence of the synchronization error on the channel noise is quite linear with a slope of 1 when $(SNR)_c > 20$ dB, as is seen in Fig. 3. When $(SNR)_c < 20$ dB, bursts of desynchronization occur to deteriorate the system performance. This means the BER depends only on $(SNR)_c$ if the channel noise is low enough so that the bit error comes from synchronization error rather than from bursts of desynchronization. However, when a large message, for example $\varepsilon = 1.0$, is transmitted, large channel noise is introduced for the same $(SNR)_c$. In this situation, the large channel noise can induce bursts of desynchronization, which dominate the bit error. When this phenomenon occurs, transmitting a message with a larger amplitude produces more error bits than transmitting a message with a smaller amplitude does for the same $(SNR)_c$.

The system performance represented as a plot of BER versus $(SNR)_c$ is shown in Fig. 4, in which each curve is obtained by fixing the message amplitude while changing the strength of the channel noise. It is clear that, for the same $(SNR)_c$, the modulation with $\varepsilon = 1.0$ generates more error bits than the modulation with $\varepsilon = 0.1$. When $\varepsilon < 0.1$, the BER does not show significant difference compared to the case with $\varepsilon = 0.1$. To compare with conventional communication systems which do not function based on the synchronization of chaos, BER is also calculated for a traditional binary phase shift keying (BPSK) system. In comparison with the BER curve obtained when $\varepsilon = 0.1$, a power penalty of 32 dB is observed. For chaotic communication systems, synchronization error and/or desynchronization burst are the major sources of bit error. Which factor dominates the bit error depends on the configuration of a system and the strength of the channel noise.

IV. Optoelectronic Feedback System

The arrangement for message encoding and decoding through additive chaos modulation in an optoelectronic feedback system is shown in Fig. 5. In this setup, the transmitter laser has a delayed optoelectronic feedback loop which generates chaotic optical pulses through a quasi-periodic route [23]. Message encoding is achieved through additive chaos modulation in the feedback loop. Though not considered in this paper, multiplicative chaos modulation is also applicable to this system [20].

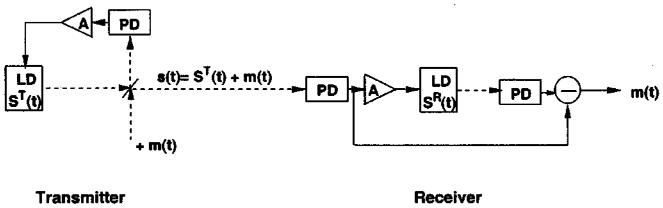

Fig. 5. Schematic setup of message encoding and decoding with additive chaos modulation in a delayed optoelectronic feedback system. Dashed lines show the optical paths.

When the encoded message, combined with the chaotic waveform from the transmitter, is used to drive the receiver, the message is fed back through the optoelectronic feedback loop to drive the dynamics of the transmitter laser. The receiver is configured as an open loop and is driven by the transmitted signal from the transmitter.

Experimental success of synchronization and message encoding/decoding in this system have been achieved and reported in [20]. In this paper, to investigate the effect of chaos modulation on laser dynamics and system BER performance, the same models for the transmitter and the receiver as in [20] are adopted, where the coupled equations of photon density S and carrier density N are used since the system is not optical phase sensitive. The transmitter laser is modeled by the following equations:

$$\frac{dS^T}{dt} = -\gamma_c S^T + \Gamma g S^T + 2\sqrt{S_0 S^T} F_s^T \quad (10)$$

$$\frac{dN^T}{dt} = \frac{J}{ed}\left[1 + \xi y^T(t-\tau)\right] - \gamma_s N^T - g S^T \quad (11)$$

$$y^T(t) = \int_{-\infty}^{t} d\eta f^T(t-\eta)\frac{s(\eta)}{S_0} \quad (12)$$

where S_0 is the photon density under the condition of free-running, F_s^T is a stochastic noise term derived from F_{sp}^T, ξ is the dimensionless feedback parameter which corresponds to the feedback strength, τ is the feedback delay time, $f^T(t)$ is the normalized response function of the feedback loop including the photodetector and the amplifier, $s(t) = S^T(t) + m(t)$ is the signal that is fed back to the transmitter and also transmitted to the receiver, and $m(t)$ is the encoded message. The other parameters are defined the same as those in Section III. Meanwhile, the received laser is modeled by

$$\frac{dS^R}{dt} = -\gamma_c S^R + \Gamma g S^R + 2\sqrt{S_0 S^R} F_s^R \quad (13)$$

$$\frac{dN^R}{dt} = \frac{J}{ed}\left[1 + \xi y^R(t-\tau)\right] - \gamma_s N^R - g S^R \quad (14)$$

$$y^R(t) = \int_{-\infty}^{t} d\eta f^R(t-\eta)\frac{s(\eta)}{S_0}. \quad (15)$$

In (10)–(15), the transmitter and receiver are identical systems even in the presence of message encoding and both are driven by the same force $s(t)$. As bandwidth limitation is not the purpose in this paper, $f^T(t) = f^R(t) = \delta(t)$ is assumed in the simulation below. With $S^R(t)$ synchronized to $S^T(t)$, message recovery is achieved by

$$m^R(t) = s(t) - S^R(t) = S^T(t) + m(t) - S^R(t) = m(t). \quad (16)$$

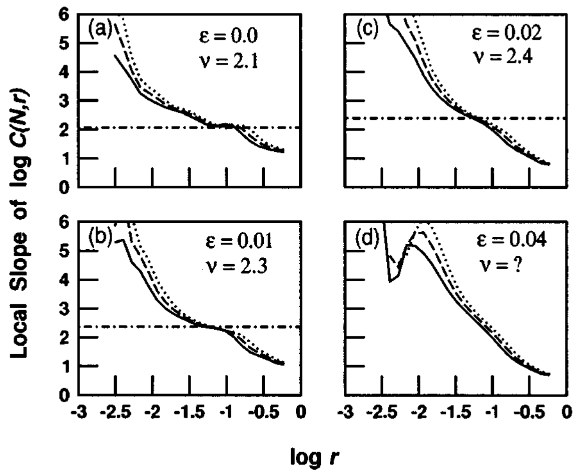

Fig. 6. Calculated correlation dimension plots with different message amplitudes in the optoelectronic feedback system.

In the delayed optoelectronic feedback system, the chaotic output of the transmitter laser is a stream of optical pulses, where the intensity of the chaotic waveform varies dramatically. To prevent the optical intensity from dropping below zero, the binary NRZ message is taken in the form as $m(t)/S_0 = 1 \pm \varepsilon$ with the "$+$" and "$-$" signs corresponding to the random bits "1" and "0," respectively. The bit rate is set at 2.5 Gb/s because the pulsing frequency of the chaotic waveform is limited by the relaxation resonance frequency of the laser around 3.5 GHz for the parameters used in the simulation. The transmission of messages at higher bit rates can be achieved with faster lasers.

The effect of message encoding on the chaotic dynamics is first investigated. From the configuration in Fig. 5, the message signal $m(t)$ is dynamically fed back to modulate the chaotic pulsing state of the transmitter laser. In this delayed optoelectronic feedback system, the correlation dimension is calculated through the peak intensity series sampled at the local maxima of $S^T(t)$ because of the pulsing characteristic of the system [23]. Fig. 6 shows the correlation dimension of the peak intensity series of the transmitter output at different signal amplitudes. In the absence of an encoded message as $\varepsilon = 0.0$ in Fig. 6(a), the correlation dimension is indicated to be $\nu = 2.1$. With a message of amplitude $\varepsilon = 0.01$, the correlation dimension ν is increased to 2.3 as shown in Fig. 6(b). When the message amplitude is increased to $\varepsilon = 0.02$ in Fig. 6(c), the plot of the correlation dimension shows no obvious flat part. We can barely tell that the correlation dimension is about $\nu = 2.4$. Finally in Fig. 6(d) when $\varepsilon = 0.04$, the correlation plot shows a characteristic of noise where the curves do not coincide with each other and have no flat parts at all. This analysis indicates that the feedback of the message into the dynamics of the transmitter increases the complexity of the chaotic state significantly even when a message of a small amplitude is encoded. The random nature of the message is very important in increasing the dimension of the chaotic state. When the message has some correlation, for example a message with repetitive "101 010," the increase of the correlation dimension of the chaotic state is found to be not as significant as the situation when the message is random. The noise-like characteristic of the chaotic state as shown in Fig. 6(d) is not observed when the message is not random. This delayed optoelectronic feedback system has the potential to generate very high-dimensional chaos when the delay time gets very long. With the combined effect of high-dimensional chaos and chaos modulation, this system can be very difficult to attack comparing with some low dimensional systems with only chaos masking.

The synchronization quality for different message amplitudes in the chaotic optical communication system with delayed optoelectronic feedback is also investigated. When there is no message encoded, the correlation coefficient between the chaotic outputs of the transmitter and the receiver is measured to be $\rho = 0.998$ and the synchronization error is $\zeta = 0.03$, where the imperfect synchronization is caused by the intrinsic spontaneous emission noise of the lasers. The synchronization quality is found to remain at the same level when the message amplitude increases from $\varepsilon = 0$ to 0.4. The message amplitude $\varepsilon < 0.4$ is necessary to keep the transmitter in the desired chaotic region. When there is parameter mismatch between the transmitter and the receiver, as what happens in real systems, both ρ and ζ are observed to deteriorate because of the mismatch, but the property of maintained synchronization quality with different message amplitudes still remains in this situation. The fact that message encoding does not influence the synchronization quality is a general property of systems with chaos modulation, which is a great advantage in chaotic communications.

In the optoelectronic feedback system, the influence of channel noise on the system performance is very important because a large amount of synchronization error can be generated by the injection of channel noise into the receiver when the recovered signal becomes $m(t) + n(t) + S^T(t) - S^R(t)$. The channel noise is injected into the receiver laser dynamics through the carrier density $N^R(t)$ when the optical intensity is converted into an electrical current by the photodetector to drive $N^R(t)$ of the receiver laser, as is shown in (14) and (15). The significance of the channel noise acting on $N^R(t)$ instead of $A^R(t)$ as in the optical injection system is that the system becomes more robust to noise with this configuration and less synchronization error is created with the same strength of channel noise. The reason is that, for a typical semiconductor laser, the carrier lifetime is generally a few orders of magnitude longer than the photon lifetime. Therefore, the relatively slowly varying carrier density $N^R(t)$ can function as a natural low-pass filter to reduce the effect of channel noise.

Even though the synchronization error can be largely reduced by using the optoelectronic feedback system, the amount of the generated synchronization error is still huge compared with the injected channel noise. Fig. 7 shows the comparison of the $(\mathrm{SNR})_c$ to the $(\mathrm{SNR})_o$ obtained at $\varepsilon = 0.2$. It is clear that $(\mathrm{SNR})_o$ is about 12 dB worse than $(\mathrm{SNR})_c$ because of the synchronization error. Different from the optical injection system, the situation of desynchronization under the influence of large channel noise is not observed in this optoelectronic feedback system for the same reason that the slowly varying $N^R(t)$ smoothes out and reduces the effect of channel noise. In Fig. 7, the linear dependence of the synchronization error on the channel noise also indicates that the BER depends only on the ratio of $(\mathrm{SNR})_c$. This property holds in the entire range of the message amplitude $\varepsilon < 0.4$. This is consistent with the requirement that $\varepsilon < 0.4$ in order to keep the output of the transmitter within the desired chaotic region. Even though linear dependence of synchronization error on channel noise is observed in the two chaotic optical communication systems demonstrated in this paper, the relationship varies from one system to another. Nonlinear dependence has also been reported in [24].

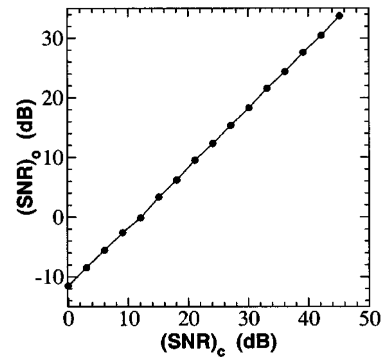

Fig. 7. Synchronization performance of the chaotice optoelectronic feedback system indicated by $(\mathrm{SNR})_c$ versus $(\mathrm{SNR})_o$.

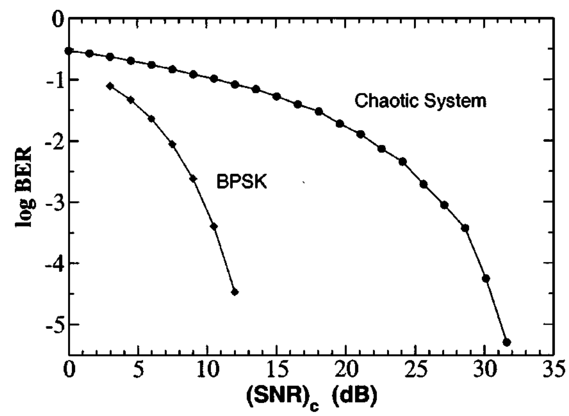

Fig. 8. BER versus $(\mathrm{SNR})_c$ of the chaotic optoelectronic feedback system. The BER of a BPSK system is also shown for comparison.

From the above discussions, the influence of channel noise on synchronization error is found to be significant. The effect of the synchronization error on the system performance depends on the characteristics of the synchronization error, which may not function as AWGN and can also be different for different systems. Fig. 8 shows the calculated BER versus $(\mathrm{SNR})_c$ under the influence of channel noise and synchronization error for the optoelectronic feedback system. In this calculation, the message amplitude is fixed at $\varepsilon = 0.2$ and the strength of the channel noise N_0 is varied. As we see, the BER drops to below 10^{-5} when $(\mathrm{SNR})_c$ exceeds 31 dB, which is a much better performance than that shown in Fig. 4 for the optical injection system. To compare with conventional communication systems, BER is also calculated for a traditional BPSK system. A power penalty of 19 dB is observed for the chaotic communication system compared with the traditional BPSK system. This power penalty is mainly caused by the enormous synchronization error generated by the channel noise. However, for chaotic communication systems, the performance can be largely improved by adding a low-pass filter at the receiver to reduce the fluctuation of channel noise and synchronization error. Other signal processing methods can also be implemented in chaotic communication systems to further improve the performance.

V. CONCLUSION

We have analyzed two chaotic optical communication systems with the implementation of the additive chaos modulation scheme for message encoding and decoding. In both the optical injection system and the optoelectronic feedback system, the influence of message encoding on the chaotic dynamics is significant. It is found that a small amount of message injected into the chaotic dynamics can increase the complexity of the chaotic state dramatically under certain situations because of the random nature of the message. This feature increases the practicality of secure communications using chaotic systems with chaos modulation.

In chaotic communication systems, channel noise does not only contaminate the transmitted signal, but also creates synchronization error in the system. This error is one of the major sources of bit error for the system. To improve the performance of a chaotic communication system, it is important that both the synchronization method and the chaotic system be robust to noise.

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